Nonlinear PET Parametric Image Reconstruction with MRI Information Using Kernel Method

Kuang Gong¹, Guobao Wang², Kevin T. Chen³, Ciprian Catana³ and Jinyi Qi*¹

¹Department of Biomedical Engineering, University of California, Davis
²Department of Radiology, University of California, Davis
³Martinos Center for Biomedical Imaging, Massachusetts General Hospital

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Dynamic PET

- Positron Emission Tomography (PET) is a highly sensitive molecular imaging modality widely used in oncology, cardiology and neurology.
- One feature of PET is its ability to capture the dynamics of radiotracer uptake in tissue.
- Dynamic PET can be useful for early lesion detection and treatment monitoring.
Parametric imaging

To reveal physiological related information, kinetic modeling was used to calculate the parametric image:

- **Resolution**: worse than that of x-ray CT and MRI due to degradation factors;
- **Noise**: limited photons received in a given scan time.

**Indirect reconstruction**

Image quality of the above framework is poor:

- **Resolution**: worse than that of x-ray CT and MRI due to degradation factors;
- **Noise**: limited photons received in a given scan time.
Opportunities and Challenges


- With the development of simultaneous PET/MRI scanners, spatially and temporally co-registered MRI information become readily available for PET image reconstruction.

**Opportunities:** use high-resolution MRI information in direct PET parametric reconstruction framework.

**Challenges:** How to include MR information and solve the corresponding optimization problem.

- Kernel representation to include MRI information
- ADMM algorithm to solve optimization problem
Dynamic Maximum likelihood reconstruction

- Denote $\theta = [f_v^T, K_1^T, k_2^T, k_3^T, k_4^T]^T$, $x_m(\theta)$ is related to $\theta$ through:

$$\begin{align*}
\frac{dC_1(t)}{dt} &= K_1 C_p(t) - (k_2 + k_3)C_1(t) + k_4 C_2(t) \\
\frac{dC_2(t)}{dt} &= k_3 C_1(t) - k_4 C_2(t)
\end{align*}$$

$$C_T(t) = (1 - f_v)[C_1(t) + C_2(t)] + f_v C_{wb}(t)$$

$$x_m(\theta) = \int_{t_{m,s}}^{t_{m,e}} C_T(\tau) e^{-\lambda \tau} d\tau$$

- Based on system model, mean of measurement data $\bar{y}_m(\theta)$:

$$\bar{y}_m(\theta) = P x_m(\theta) + r_m$$

- Likelihood function for dynamic PET:

$$L(y|\theta) = \sum_{m=1}^{M} \sum_{i=1}^{N_s} y_{im} \log(\bar{y}_{im}(\theta)) - \bar{y}_{im}(\theta)$$
Prior Art: Direct Reconstruction

- **Optimization Transfer** with **EM surrogate (OTEM) algorithm** (Wang and Qi 2012) is a **monotonic** direct reconstruction method for nonlinear models:

\[
L(y|\theta) = \sum_{m=1}^{M} \sum_{i=1}^{N_s} y_{im} \log(\bar{y}_{im}(\theta)) - \bar{y}_{im}(\theta)
\]

\[
Q(\theta; \theta_n) \leq L(y|\theta)
\]

\[
Q(\theta_n; \theta_n) = L(y|\theta_n)
\]

Optimization transfer

\[
Q(\theta; \theta_n) = \sum_{j=1}^{N} \left( \sum_{i=1}^{N_s} p_{ij} \right) \left[ \sum_{m=1}^{M} \hat{x}_{jm}^{em,n} \log x_{jm}(\theta_j) - x_{jm}(\theta_j) \right]
\]

\[
\hat{x}_{jm}^{em,n} = \frac{x_m(\theta^n_j)}{\sum_{i=1}^{N_s} p_{ij} \sum_{i=1}^{N_s} \frac{y_{im}}{y_{im}(\theta^n)}}
\]

Step 1

Time activity curve (TAC) fitting pixel by pixel based on objective function \( Q(\theta; \theta_n) \). **Step 2**
Kernel Representation

- **Static PET:** \( x = K \alpha \)

- **Parametric imaging:**

  \[
  f_v = K \alpha_1 \\
  K_1 = K \alpha_2 \\
  k_2 = K \alpha_3 \\
  k_3 = K \alpha_4 \\
  k_4 = K \alpha_5
  \]

  \( \theta = [f_v^T, K_1^T, k_2^T, k_3^T, k_4^T]^T \)

  \( \alpha = [\alpha_1^T, \alpha_2^T, \alpha_3^T, \alpha_4^T, \alpha_5^T]^T \)

  \( \theta = (I_5 \otimes K) \alpha \)

- Radial Gaussian kernel

\[ K_{ij} = \exp\left(-\frac{||\mu_i - \mu_j||^2}{2\sigma^2}\right) \]
Challenges in Optimization

• Based on Kernel representation,

\[ \theta = (I_5 \otimes K) \alpha \]

• This kernel representation cannot be embedded into OTEM framework as the coefficients \( \alpha \) are coupled by the kernel matrix, and pixel by pixel TAC fitting is not possible.

• Transfer to constrained problem:

\[
\begin{align*}
\max & \quad L(y|\theta) \\
\text{s.t.} & \quad \theta = (I_5 \otimes K) \alpha.
\end{align*}
\]
ADMM

\[ \max \quad L(y|\theta) \]
\[ \text{s.t.} \quad \theta = (I_5 \otimes K) \alpha. \]

\( \rho \) : penalty parameter

\( \mu \) : scaled dual variable

Augmented Lagrangian format

\[ L_\rho = L(y|\theta) - \frac{\rho}{2} ||\theta - (I_5 \otimes K) \alpha + \mu||^2 + \frac{\rho}{2} ||\mu||^2. \]

Alternating direction of multiplier method (ADMM)

\[
\begin{align*}
\theta^{n+1} &= \arg\max_{\theta} L(y|\theta) - \frac{\rho}{2} ||\theta - (I_5 \otimes K) \alpha^n + \mu^n||^2 \\
\alpha^{n+1} &= \arg\min_{\alpha} ||(I_5 \otimes K) \alpha - (\theta^{n+1} + \mu^n)||^2 \\
\mu^{n+1} &= \mu^n + \theta^{n+1} - (I_5 \otimes K) \alpha^{n+1}.
\end{align*}
\]
**Sub-problem 1:** \( \theta^{n+1} = \arg\max_{\theta} L(y|\theta) - \frac{\rho}{2} ||\theta - (I_5 \otimes K) \alpha^n + \mu^n ||^2 \)

This is a direct reconstruction with quadratic penalty problem. Solved using modified OTEM algorithm mentioned previously.

**Sub-problem 2:** \( \alpha^{n+1} = \arg\min_{\alpha} ||(I_5 \otimes K) \alpha - (\theta^{n+1} + \mu^n) ||^2 \)

This is a least squares problem. Solved using conjugate gradient algorithm.
2D Simulation

TAC for different tissues

- Brainweb phantom, mCT geometry
- 128 x 128, 2 x 2 mm$^2$
- Scanning protocol, 24 frames in 1 hour: 4 x 20 s, 4 x 40 s, 4 x 60 s, 4 x 180 s, 8 x 300 s

MR prior image

Ground truth of $K_i$

Table 1: The simulated kinetics

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$f_v$</th>
<th>$K_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>White matter</td>
<td>0.02</td>
<td>0.046</td>
<td>0.080</td>
<td>0.050</td>
<td>0.001</td>
</tr>
<tr>
<td>Grey matter</td>
<td>0.03</td>
<td>0.071</td>
<td>0.090</td>
<td>0.060</td>
<td>0.002</td>
</tr>
<tr>
<td>Tumor</td>
<td>0.05</td>
<td>0.082</td>
<td>0.055</td>
<td>0.090</td>
<td>0.003</td>
</tr>
</tbody>
</table>

$K_i = \frac{K_1 k_3}{k_2 + k_3}$

$K_i$ : Proportional to glucose metabolism rate
Simulation Results for $K_i$

- Kernel method can reduce noise and recover the tumor.

Indirect • OTEM + Nonlocal Mean • Kernel
• Tumor and white matter were chosen to compute CRC-STD quantification.
• Kernel method is better than the other methods.
Patient Data Acquisition

- Siemens Brain PET-MR scanner, 5 mCi FDG injection
- 70 min dynamic scan with 21 frames: 4 x 30 s, 2 x 60 s, 2 x 150 s, 2 x 180 s, and 11 x 300 s.
- MRI guided attenuation and motion correction.
- Blood input function obtained from reconstructed PET images using vessel region drawn from T1 weighted MRI.
Patient $K_i$ image

MRI prior  OTEM + filter  OTEM + Nonlocal Mean  Kernel
Summary and Future Work

- MR information was included into direct parametric reconstruction based on two tissue compartment model using kernel representation;
- ADMM algorithm was used to solve the constrained problem;
- Evaluation of $K_i$ images in both simulation and real data shows the benefits of the proposed method;
- Further evaluations are needed using real datasets.

Thanks for your attention!